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IDENTIFICATION OF SPHERE SIZE USING THE PHANToM™: TOWARDS A SET OF BUILDING BLOCKS FOR RENDERING HAPTIC ENVIRONMENT

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ABSTRACT

This paper presents an approach to developing building blocks for rendering synthetic haptic environments using force-reflective haptic interfaces. The task of sphere size identification is used to explore an accurate and efficient way of estimating human performance using the absolute identification paradigm. The results show that subjects can reliably identify 3 to 4 sphere sizes with a set of spheres ranging from 10 to 80 mm in radius. Subjects with prior experience with force-reflective haptic interfaces exhibit higher performance levels. The results also indicate that gradually increasing stimulus complexity yields more accurate measurements of information transmission at little additional cost in terms of total number of experimental trials. Subsequent studies will utilize the new experimental protocol developed in this study.

1. INTRODUCTION

Much progress has been made in the recent years in the area of haptic human-machine interfaces, and much work still remains (e.g., see reviews by Srinivasan, 1994; and Burdea, 1996). Haptic interfaces are devices that enable manual interactions with synthetic environments or teleoperated robotic systems. Compared with visual and auditory systems, haptic interfaces are unique in the sense that they are both *displays* and *controllers*. The basic parameters that are displayed or used for control are force and position (and their spatial and temporal distributions), through which the human user experiences a synthetic haptic environment (Fig. 1). This paper is mainly concerned with the use of haptic interfaces as *displays* of synthetic haptic environments.

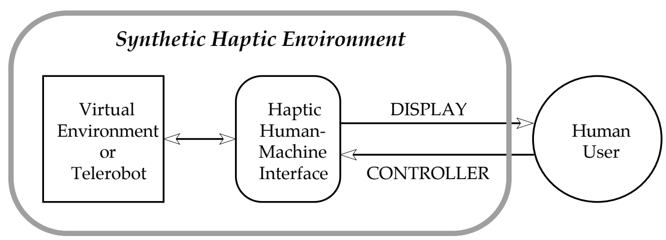


Figure 1. Haptic human-machine interfaces are both displays and controllers.

To demonstrate that a haptic display is effective, quantitative human studies at several levels are essential. The three frequently used psychophysical paradigms are *detection*, *discrimination*, and *identification*. Absolute detection thresholds obtained from detection studies reveal the smallest parameter values that a human can perceive. Differential thresholds obtained from discrimination studies reveal the smallest perceivable differences in parameter values between a reference and a test object (see Gescheider, 1985 for a reference on these paradigms). When detection and discrimination experiments are conducted properly, results are not dependent on test apparatus. Knowledge of detection and discrimination thresholds can therefore guide the hardware design specifications for haptic human-machine interfaces in general (see, e.g., Tan, Srinivasan, Eberman, & Cheng, 1994).

Absolute identification paradigms measure a human observer's ability to categorize parameter values without providing explicit references. If the human haptic sensory system is viewed as a noisy communication channel, then identification experiments reveal the capacity of such a channel in transmitting haptic information. While detection and discrimination performance is limited mainly by sensory noise in the peripheral somatosensory system, identification performance is also limited by memory noise (e.g., see Durlach & Braida, 1969 for a model and data in the area of auditory intensity perception). Therefore, the absolute identification paradigm is more appropriate for evaluating how well human users perform with haptic interfaces. In addition, because results of the absolute identification paradigm are expressed in *bits* of information transferred between a human operator and a device, it is possible to make direct performance comparison of different haptic interfaces and displays for different modalities. For example, in his classical paper, Miller (1956) concluded that identification performance is limited to 2–3 *bits* (i.e., people can reliably identify about 7±2 categories) when a human observer is asked to attend to changes in one parameter, regardless of the sensory modalities involved.

There are many examples in daily lives where people do much better than 2–3 *bits* (e.g., face recognition). The key is to employ many parameters that are perceptually distinctive and yet do not interfere too much with one another when they are combined. Recently, Tan (1996) used the absolute identification paradigm as a tool to explore ways of achieving optimum information transmission with a three-finger position display called the Tactuator. It was found that as much as 6.5 *bits* (corresponding to perfect identification of 90 movement patterns of three fingers) can be achieved by constructing stimuli from a set of carefully chosen "building blocks". These building blocks were slow motion, fluttering/rough motion, vibration, their associated frequencies and amplitudes, stimulation site and direction of motion. The same technique can be used to identify the *haptic primitives* that a force display can render in creating a synthetic haptic environment.

The overall goals of the current and subsequent studies are (1) to explore the various haptic features (e.g., size and stiffness) that give rise to distinctive perception, (2) to establish baseline performance levels for identifying values of single features in isolation (or with values of other features held constant), (3) to measure the amount of perceptual interference when multiple features are combined, and finally (4) to develop a set of "building blocks" for rendering haptic objects based on these features and their interaction. Much work has been accomplished in the past in the area of using position displays to deliver speech information to people with hearing or visual impairments (see Summers, 1992 and Geldard, 1973). A significant challenge in the general use of these devices is the amount of training that is required for the user to learn to associate the often unfamiliar haptic sensations with meaningful messages or events. Force-reflective devices have the advantage of delivering information that feels "natural" to novice users. It is hoped that an understanding of the salient haptic features will have a significant impact on the way haptic rendering software systems are designed.

This paper can be viewed as a method validation for subsequent work. The current study uses the example of sphere size identification to examine how to accurately and efficiently collect data with human observers using the absolute identification paradigm. The task of identifying sphere size was chosen because size is a salient feature of any object, and a sphere is a three-dimensional object whose size can be characterized with only one parameter, namely its radius. Section 2 introduces the absolute identification paradigm and issues related to the accuracy and efficiency of estimating information transfer. A new approach to choosing stimuli is presented. Section 3 describes the methodology of this study. Section 4 summarizes the experimental results. Finally, a general discussion is presented in Section 5.

2. ABSOLUTE IDENTIFICATION PARADIGM

The absolute identification paradigm involves a set of k stimuli S_i, $1 \le i \le k$, a set of k responses, R_j, $1 \le j \le k$, and a one-to-one mapping between the stimuli and responses. The stimuli are presented one at a time in random order with equal *a priori* probabilities and the subject is instructed to respond to each stimulus presentation with the response defined by the one-to-one mapping, i.e., to identify which of the k stimuli was presented. Without loss of generality, it is assumed that R_i is the label for S_i under the one-to-one mapping. In other words, R_i is the correct response to S_i. In general, identification performance may depend not only on the characteristics of the stimulus set, but also on the extent to which the mapping between stimuli and responses are "compatible". (The issue of "stimulus-response compatibility", however, is beyond the scope of this paper. Interested readers may refer to Proctor & Reeve, 1990). Results of absolute identification experiments are usually summarized in terms of the first-order stimulus-response matrix, i.e., the $k \times k$ matrix in which the entry in row i and column j specifies the number of times stimulus S_i led to response R_j. In other words, it is assumed that the trials are statistically independent and all possible sequential effects are ignored.

The Concept of Information and Its Estimate

The data processing scheme for an absolute identification paradigm adapts concepts and mathematics from information theory (see Cover & Thomas, 1991 for an introduction to information theory; see Garner, 1962 and Quastler, 1954 for applying information theory to Psychology). Information is something we get when we learn something we didn't know before. Any communication act provides information only insofar as it reduces a condition of ignorance or uncertainty about the state of things under consideration. Information transfer (IT) measures the increase in information about the signal transmitted resulting from knowledge of the received signal. For a particular stimulus-response pair (S_i, R_j), IT is given by $\log_2\left[P\left(S_i/R_j\right)/P\left(S_i\right)\right]$, where $P\left(S_i/R_j\right)$ is the conditional probability of S_i given R_j, and $P\left(S_i\right)$ is the *a priori* probability of S_i. The average information transfer (IT) is thus given by

$$IT = \sum_{j=1}^{k}\sum_{i=1}^{k} P(S_i, R_j)\log_2\left(\frac{P\left(S_i/R_j\right)}{P\left(S_i\right)}\right), \tag{1}$$

or, equivalently

$$IT = \sum_{j=1}^{k}\sum_{i=1}^{k} P\left(S_i, R_j\right)\log_2\left(\frac{P\left(S_i, R_j\right)}{P\left(S_i\right)P\left(R_j\right)}\right), \tag{2}$$

where $P(S_i, R_j)$ is the joint probability of stimulus S_i and R_j, and $P\left(R_j\right)$ is the probability of R_j.

A related quantity, 2^{IT}, is interpreted as the number of stimulus categories that can be correctly identified. It is an abstract concept since 2^{IT} is not necessarily an integer.

The maximum likelihood estimate of IT, IT_{est}, from a first-order stimulus-response matrix is computed by approximating underlying probabilities with frequencies of occurrence:

$$IT_{est} = \sum_{j=1}^{k}\sum_{i=1}^{k}\frac{n_{ij}}{n}\log_2\left(\frac{n_{ij}\cdot n}{n_i \cdot n_j}\right), \tag{3}$$

where n is the total number of trials collected, n_{ij} is the number of times the joint event (S_i, R_j) occurs, and $n_i = \sum_{j=1}^{k} n_{ij}$ and $n_j = \sum_{i=1}^{k} n_{ij}$ are the row and column sums. Unfortunately, IT_{est} is not only subject to statistical fluctuations, but it is also a biased estimate: it tends, for a limited number of trials, to overestimate IT. Further, the magnitude of the bias tends to greatly exceed the magnitude of the fluctuations (Rogers & Green, 1954; Rabinowitz, Houtsma, Durlach, & Delhorne, 1987; Houtsma, 1983). Two approaches have been tried to correct for the bias in IT_{est} given limited amount of experimental data. Miller (1954) derived a correction formula, $(k-1)^2 \cdot (\log_2 e)/(2n)$, based on the first-order difference between IT and the expected value of IT_{est}. When performance level is high and n is not significantly larger than k^2 (e.g., $n < 5k^2$), Miller's formula tends to over-correct. An extreme case is that when performance is perfect (therefore no correction is needed), the correction term is nonzero unless $n >> k^2$. Houtsma (1983) presented a computer-simulated approach to derive unbiased estimates of mutual information for a tactile identification task (see Rabinowitz et al., 1987) with $k = 125$ and $n = 5{,}000$. Responses were modeled as $r = s + w$, where w was a uniformly distributed random integer in the range $-l \le w \le l$, $l \le k$. Estimated IT vs. n curves for different l values were generated from simulated stimulus-response confusion matrices, and the maximum n was sufficiently large for the curves to reach asymptotic levels. Empirically determined IT_{est} vs. n curve was fit to the nearest simulated function and the asymptotic value of the simulated curve was read off as the unbiased estimate of IT. One interesting finding in Houtsma (1983) was that when l was very small (i.e., performance level was high), IT_{est} converged quickly to its asymptotic value. As l increased (i.e., performance degraded), more samples were needed before IT_{est} asymptoted.

Issues in Designing an Absolute Identification Experiment

Five issues need to be addressed in designing an absolute identification experiment. First, the range of parameter(s) need to be determined. According to Durlach & Braida (1969) and Braida & Durlach (1972), IT increases as parameter range increases until it reaches its asymptotic value. Therefore, whenever possible, the largest range should be used in order to obtain the highest IT. Second, number of stimulus alternatives k needs to be determined. It is generally accepted that IT equals $\log_2 k$ (also called *stimulus uncertainty* when all stimuli are presented with equal *a priori* probabilities) when k is very small (e.g., $k = 2$), increases monotonically with k, and asymptotes when k is large. Therefore, attempts to determine the maximum IT (i.e., the plateau level) usually involve selecting a k such that $\log_2 k$ is large (by 1 or 2 *bits*) relative to expected IT. A larger k, however, usually requires more trials to be collected. The third issue concerns spacing between the k stimulus alternatives. Ideally, adjacent stimuli should be separated by equal number of just-noticeable-differences (see Gescheider, 1985). This means logarithmic spacing when Weber's law applies (see Gescheider, 1985). Another commonly used method is linear spacing assuming that perceptual sensitivity increases linearly with the difference in parameter value. The ideal spacing between stimuli can only be determined by discrimination studies. The fourth issue concerns how to determine the total number of trials to be collected, n. A general rule of thumb is to collect at least $5k^2$ number of trials. This can quickly become formidable if k is large. Finally, a training procedure needs to be set up and criteria for the termination of training need to be determined beforehand.

Tan (1996) found that the relationship between IT and k was not necessarily monotonic. Therefore, selecting one large k may not reveal the maximum IT that can be achieved by a human observer. In this study, subjects were tested with a few k values such that the identification task went from trivial to difficult. This approach was aimed at obtaining an accurate estimate of IT with manageable amount of time required of the experimenter and human subjects. To begin with, $k = 2$ and $n = 5k^2 = 20$. This short, trivial task served to familiarize subjects with experimental procedures in general. In each subsequent test, k was doubled until percent-correct scores were significantly lower than 100%, which corresponded to a saturation in IT.

3. GENERAL METHODS

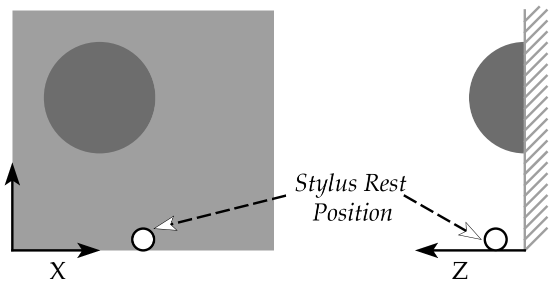
Apparatus

The PHANToM™ (SensAble Technologies Inc., Cambridge, MA) was used to deliver stimuli in the current study. The PHANToM (Personal HAptic iNTerface Mechanism) is a point-contact force-reflective device that monitors the 3D position of the tip of a thimble or a stylus and applies a three-dimensional force-vector that is appropriate for the synthetic haptic environment it renders (Massie, 1996). The model used in this study (ver. 1.0a) is equipped with a stylus with an activation switch.

Stimulus

After examining the work space of the PHANToM, it was decided that a hemisphere on a vertical wall would be generated for two reasons (Fig. 2). First, as noted in Massie (1996, "Feeling Through an Object", p. 17), when the human observer uses a stylus to feel the back side (i.e., the side away from the observer) of a simulated sphere, the user can only feel a force at the tip of the stylus although part of the stylus may be inside the simulated sphere. A novice user may be distracted by this "unnatural" effect. Second, the work space is about twice as wide (in X direction) and tall (in Y direction) as it is deep (in Z direction). Therefore, a hemisphere on a vertical wall produces the largest radius range within the available work volume. The plane of the wall was kept at a fixed distance of 30 mm from the stylus rest position. The radius values of the smallest and largest hemispheres were 10.0 mm and 80.00 mm, respectively. For all k values, radius range was kept at its maximum. Both linear and logarithmic spacings were used for generating stimuli.

To calibrate the size of a stimulus, a pencil head was attached to the end of the stylus. The user traced the great circle where the



(a) Front view. (b) Side view.

Figure 2. Illustration of a hemisphere on a vertical wall.

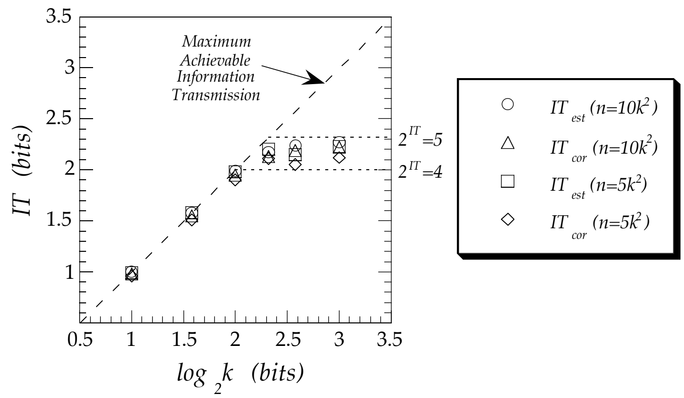


Figure 3. Estimated information transfers for subject S1.

hemisphere intersects the plane of the wall. By placing a pad slightly behind the wall, it was possible to get a drawing of the great circle. The radius of the circle was then measured to estimate the *perceived* radius of the hemisphere. It was found that the estimates were within 2 mm of intended radius values. Considering the fact that human fingerpads can easily yield by 2 mm when grasping a physical object, this was considered to be acceptable.

Subject

Six subjects, three females (S1, S3 and S4) and three males (S2, S5 and S6), participated in this study as unpaid volunteers. They were divided into two groups. Subjects S1 to S3 were tested with linearly-spaced stimuli (e.g., 10.0, 45.0 and 80.0 mm), while subjects S4 to S6 were tested with logarithmically-spaced ones (e.g., 10.0, 28.28 and 80.0 mm). Among the subjects, S1, S4 and S6 are experienced users of force-reflective devices.

Procedure

Each subject was trained and tested with a set of synthetic hemispheres on a synthetic vertical wall. The locations of the hemispheres on the wall were randomized within roughly a 200 mm by 200 mm area (see Fig. 2a). During training, the subject entered a number between 1 and k (number of radius values in the stimulus set), and the PHANToM simulated a synthetic hemisphere with the corresponding size. During testing, the standard absolute identification paradigm was used. Subjects used integers between 1 and k as responses (with 1 corresponding to the smallest radius value and k to the largest). The radius values of the smallest and largest hemispheres were always kept the same (i.e., $S_1 \equiv 10mm$ and $S_k \equiv 80mm$, respectively). All subjects were trained and tested at $k = 2,4,8$ and $n = 5k^2$. Extra data were taken with S1 so that this subject was trained and tested at $k = 2,3,4,5,6,8$ and $n = 10k^2$. No trial-by-trial correct-answer feedback was provided at any time. For both training and testing, subjects were always informed of the approximate location of the hemisphere center relative to the stylus resting position before each trial (e.g., 3 inches to the right and 2 inches up) so that they could find the hemisphere haptically with least effort. This information was especially useful when very small hemispheres were presented.

During the experiment, the subject sat in dark in front of the PHANToM with the right hand holding onto the stylus of the PHANToM, and the left hand typing responses on a keyboard. The monitor was placed to the left of the subject's body, such that the subject looked away from the PHANToM during the experiments.

4. RESULTS

Data for subject S1 are summarized in terms of IT_{est}, IT_{cor} and percent-correct scores as functions of $\log_2 k$, where $IT_{cor} = IT_{est} - (k-1)^2 \cdot (\log_2 e)/(2n)$ is the corrected estimate of IT using Miller's (1954) formula. Fig. 3 shows estimated and corrected IT for the initial $5k^2$ trials and for all $10k^2$ trials. In general, estimated IT increases with $\log_2 k$ and asymptotes at around $k = 5$. IT_{est} for the initial $5k^2$ trials and for all $10k^2$ trials differ by less than 0.1 *bit* for all k values. Therefore, it seems clear (at least for S1) that $5k^2$ trials are sufficient. The correction term varies from 0.03 to 0.11 bits for $n = 5k^2$ and 0.02 to 0.05 for $n = 10k^2$, and therefore is not significant compared with the asymptotic IT values. As mentioned before, IT_{cor} tends to be over-corrected when performance level is high. This is evident in the cases of $k = 2,3,4$ when S1's percent-correct scores are 100% (see also Fig. 4). Although it might be theoretically important to consider the differences among the four measurements shown in Fig. 3, the practical goal here is to determine the maximum number of radius values that can be perfectly identified within a radius range of 10 to 80 mm. In other words, only IT values that correspond to *integer* 2^{IT} values are relevant. The two horizontal lines in Fig. 3 correspond to IT values of 2 and 2.32 *bits* (2^{IT}=4 and 5), respectively. Since the IT_{est} and IT_{cor} values at $k = 5,6,8$ are all within this IT range, it is clear that S1 can perfectly identify at most 4 radius values. This result is further supported by the fact that S1 did perform perfectly when $k = 4$.

From Fig. 4, it is clear that percent-correct scores are 100% when k is very small and decrease as k increases. Although percent-correct scores cannot serve as a parsimonious measure of identification performance like an asymptotic IT_{est} can, it can be

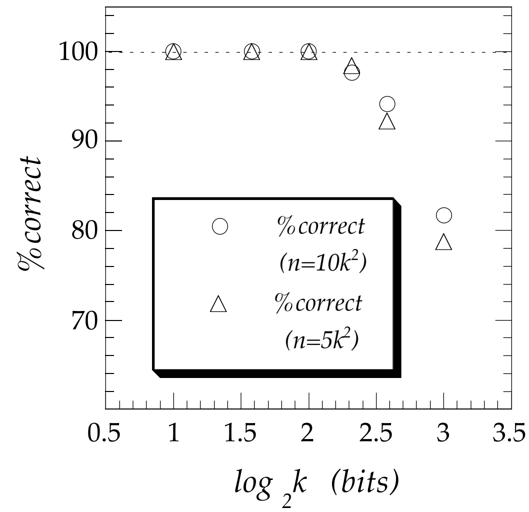


Figure 4. Percent-correct scores for subject S1.

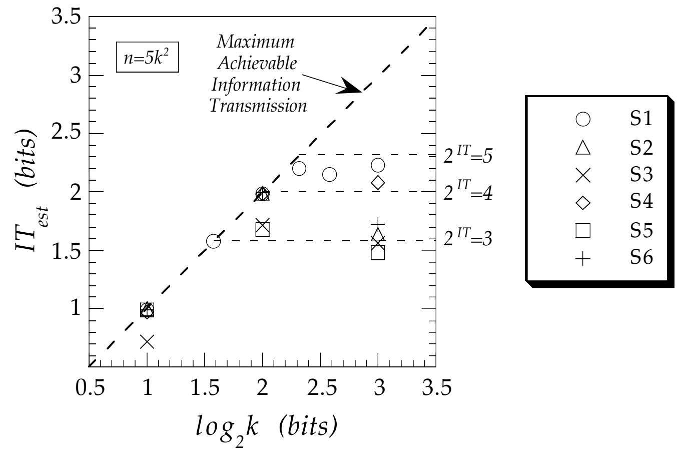


Figure 5. Estimated information transfers for all subjects.

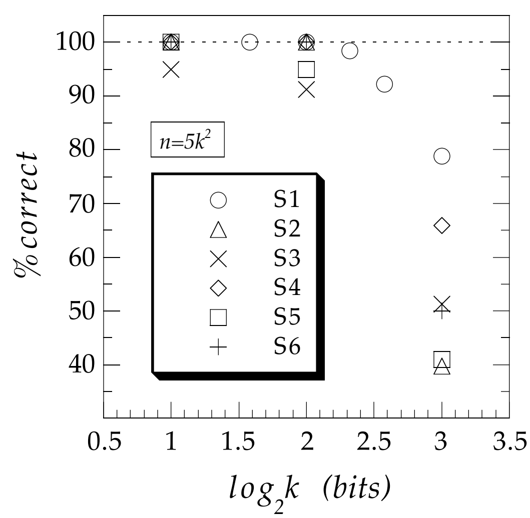


Figure 6. Percent-correct scores for all subjects.

used as an empirical indication of whether subject's performance in terms of information transmission is saturated. For example, as can be seen in Fig. 4, percent-correct scores below 95% (i.e., for $k \geq 6$) correspond to IT values significantly away from the diagonal maximum transmission line in Fig. 3.

The IT_{est} values for all subjects are shown in Fig. 5. The horizontal lines indicate performance levels corresponding to 3, 4 and 5 perfectly-identified categories. If only the data points at $k = 8$ are considered (which would be the case with the conventional experimental method of selecting one large k value), then it can be concluded that S1 and S4 can correctly identify 4 radius values, S2 and S6 can correctly identify 3 radius values, and S3 and S5 can correctly identify at most 2 radius values. Prior experience with force-reflective haptic interfaces (as in the case for S1, S4 and S6) results in better performances. An examination of S2 and S6's data at $k = 4$, however, reveals that both subjects performed perfectly with 4 radius values. S3 and S5's data at $k = 4$ indicates that these subjects can correctly identify 3 radius values. In other words, subjects S2, S3, S5 and S6 achieved higher IT_{est} scores at $k = 4$ than at $k = 8$. The IT_{est} vs $\log_2 k$ relationship for these subjects were, therefore, non-monotonic. For these four subjects, the maximum, not the asymptotic, value of IT_{est} should be taken as the correct measure of subject's perceptual capacity. At $k = 2$, all subjects performed perfectly except for S3. An interview with S3 after the experiments revealed that this subject sometimes confused the force due to mechanical limits at the workspace boundary with that simulated for synthetic objects. This may explain why this subject was not even able to distinguish a pair of spheres with radius values as different as 10 and 80 mm.

In summary, all subjects who are experienced with force-reflective displays (S1, S4 and S6) as well as one inexperienced subject (S2) can correctly identify at most 4 sphere sizes. One inexperienced subject (S5) can correctly identify at most 3 sphere sizes. It is not clear if S3's performance level would have been higher if this subject have had more experience with force-reflective haptic displays in general.

Fig. 6 shows the percent-correct scores for all subjects. It is interesting to note that a higher percent-correct score does not necessarily result in higher IT_{est}. This is because that percent-correct scores only look at the main diagonal cells in the $k \times k$ stimulus-response matrix whereas IT_{est} measures also take into account the spread and consistency of non-diagonal cells.

As mentioned above, subjects S1 to S3 were assigned linearly-spaced stimulus sets and the others were assigned logarithmically-spaced ones. It seems that this does not have a significant impact on performance levels since the two subjects who achieved highest IT_{est} scores (S1 and S4 in Fig. 5) are from both groups.

Subjects used different strategies in judging the size of the hemispheres simulated in this study. Although judging the radius of the great circle where a hemisphere joins the vertical wall should yield (intuitively) the most accurate estimate, not all subjects relied solely on circling the great circles. According to anecdotal notes, S1 rarely felt the great circles. This subject followed the convex semisphere surface to get a feeling for the curvature of that surface. S2 tried to gauge the protrusion of the hemisphere in the Z direction in addition to feeling the great circles. S4 used the size of a ping-pong ball, an orange, etc. as perceptual anchors for judging the size of hemispheres.

5. DISCUSSION

This paper describes a first experiment aimed at defining a set of building blocks for rendering synthetic haptic environments — sphere size identification. It is concluded that human observers can correctly identify at most 3 to 4 sphere sizes for spheres ranging from 10 to 80 mm in radius. Prior experience with force-reflective haptic interfaces results in better performances. This result is quite consistent with that of manual length identification reported by Durlach, Delhorne, Wong, Ko, Rabinowitz, & Hollerbach (1989). Durlach et al. (1989) obtained an information transfer of 2 *bits* (corresponding to perfect identification of 4 lengths) for broad-range (90 mm) stimulus sets and that of roughly 1 *bit* (corresponding to perfect identification of 2 lengths) for small-range (18 mm) stimulus sets. The results of this study and those of Durlach et al. (1989) are consistent with those found for other stimulus parameters and other sensory modalities.

A new procedure of selecting k was introduced in this study. It was found that for some subjects, IT_{est} peaked at $k=4$ and dropped at $k=8$. Thus it seems that selecting a few k values (instead of selecting one large k value) is a more accurate way of estimating *maximum* IT. It is an efficient method as well. The total number of trials each subject (except S1) performed was $5 \cdot 2^2 + 5 \cdot 4^2 + 5 \cdot 8^2 = 420$, compared to 405 trials for $k=9$ and 500 trials for $k=10$. Moreover, subjects' performance levels are usually high at $k=2$ and $k=4$, so that they are more motivated during these sessions. Because the difficulty of the tasks increased gradually, training effects at higher k values are also minimized.

It is not clear why the relationship between IT_{est} and k is monotonic for some subjects and exhibits a peak for others. If a subject's response is modeled as a normally-distributed noise variable, then the IT_{est} vs k curve should always be monotonic. It is possible that different amount of memory noise are present for low and high k values. This is one issue that needs further investigation and modeling. Another unresolved issue concerns spatial distortion of haptic percepts. Fasse, Hogan, Kay & Mussa-Ivaldi (1997) found that a human observer's perception of length, angle and orientation are distorted. In the current study, one subject commented that spheres felt differently depending on their distance from the subject's body.

Future studies will follow the goals outlined in the Introduction section of this paper. The new experimental protocol of gradually increasing stimulus complexity will be used. The ultimate goal is to develop a set of haptic primitives for rending synthetic haptic environments.

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